

# Week 4 Video 4

Knowledge Inference:  
Item Response Theory

# Item Response Theory

A classic approach for assessment, used for decades in tests and some online learning environments

In its classical form, has some key limitations that make it less useful for assessment in online learning

But variants such as ELO and CDM address some of those limitations

# Key goal of IRT

Measuring how much of some latent trait a person has

How intelligent is Bob?

How much does Bob know about snorkeling?

SnorkelTutor

# Typical use of IRT

Assess a student's current knowledge of topic X

Based on a sequence of items that are *dichotomously scored*

E.g. the student can get a score of 0 or 1 on each item

# Key assumptions

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There is only one latent trait or skill being measured per set of items

This assumption is relaxed in the extension  
Cognitive Diagnosis Models (CDM)  
(Henson, Templin, & Willse, 2009)

No learning is occurring in between items

E.g. a testing situation with no help or feedback

# Key assumptions

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Each learner has ability  $\theta$

Each item has difficulty  $b$  and discriminability  $a$

From these parameters, we can compute the probability  $P(\theta)$  that the learner will get the item correct

# Note

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The assumption that all items tap the same latent construct, but have different difficulties

Is a very different assumption than is seen in PFA or BKT

# The Rasch (1PL) model

Simplest IRT model, very popular

Mathematically the same model (with a different coefficient), but some different practices surrounding the math (that are out of scope for this course)

There is an entire special interest group of AERA devoted solely to the Rasch model (RaschSIG) and modeling related to Rasch



# The Rasch (1PL) model

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No discriminability parameter

Parameters for student ability and item difficulty

# The Rasch (1PL) model

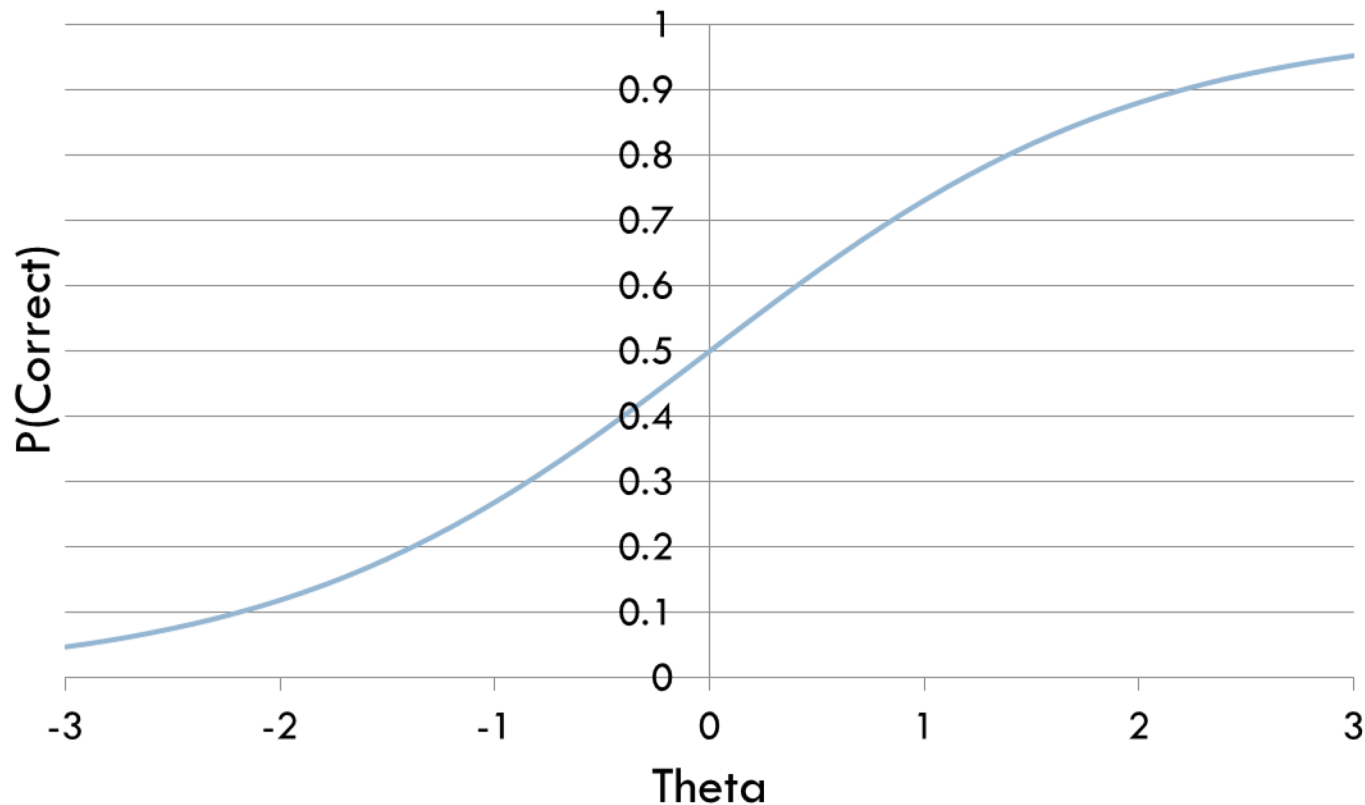
Each learner has ability  $\theta$

Each item has difficulty  $b$

$$P(\theta) = \frac{1}{1 + e^{-1(\theta - b)}}$$

# Item Characteristic Curve

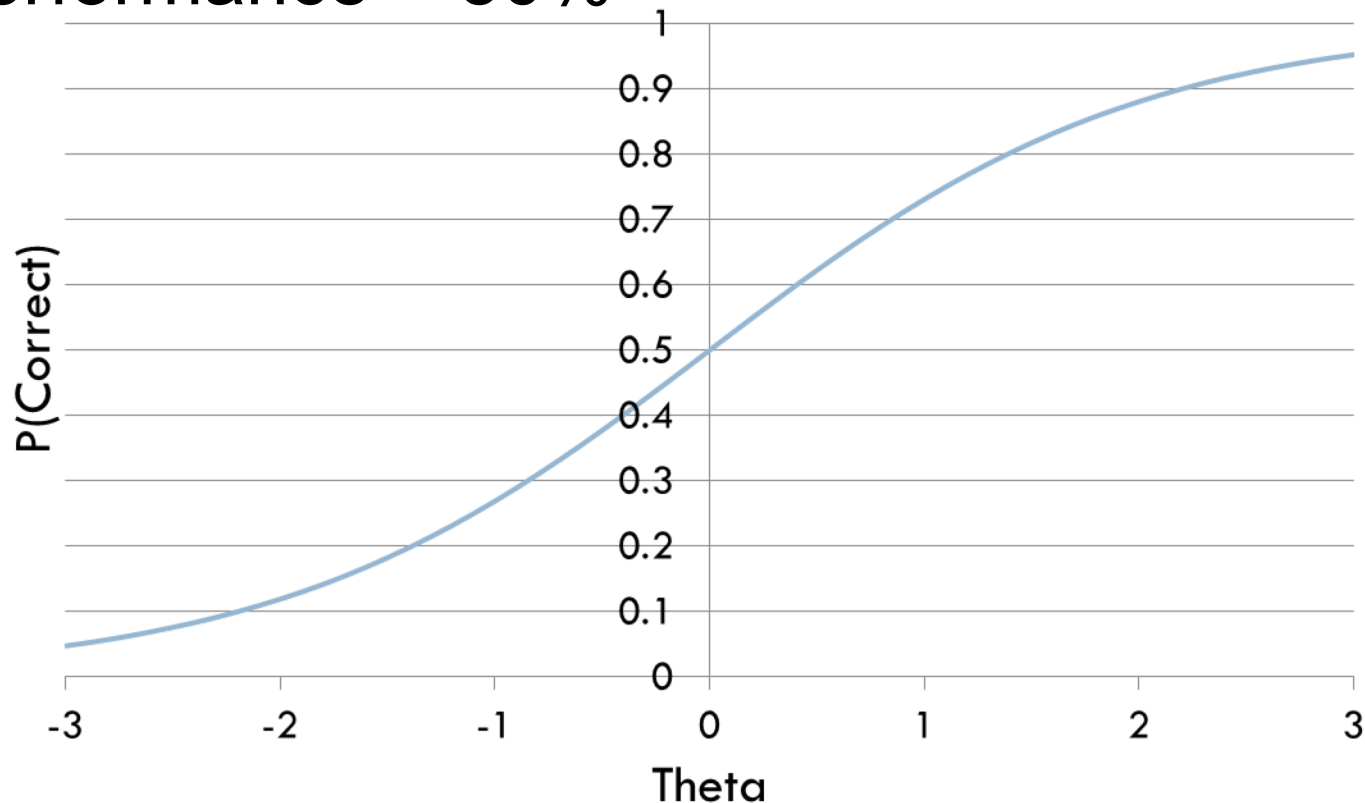
A visualization that shows the relationship between student skill and performance



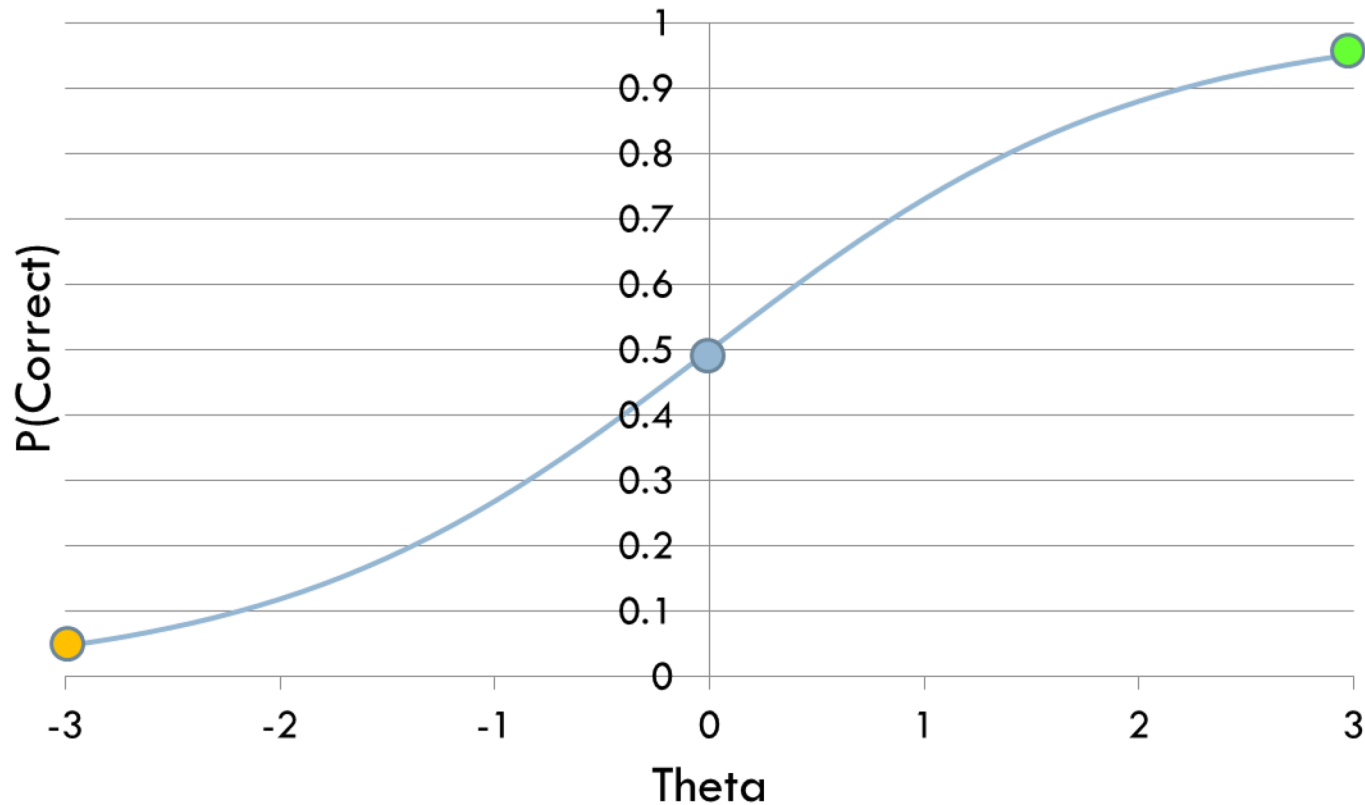
# As student skill goes up, correctness goes up

This graph represents  $b=0$

When  $\theta=b$  (knowledge=difficulty),  
performance = 50%



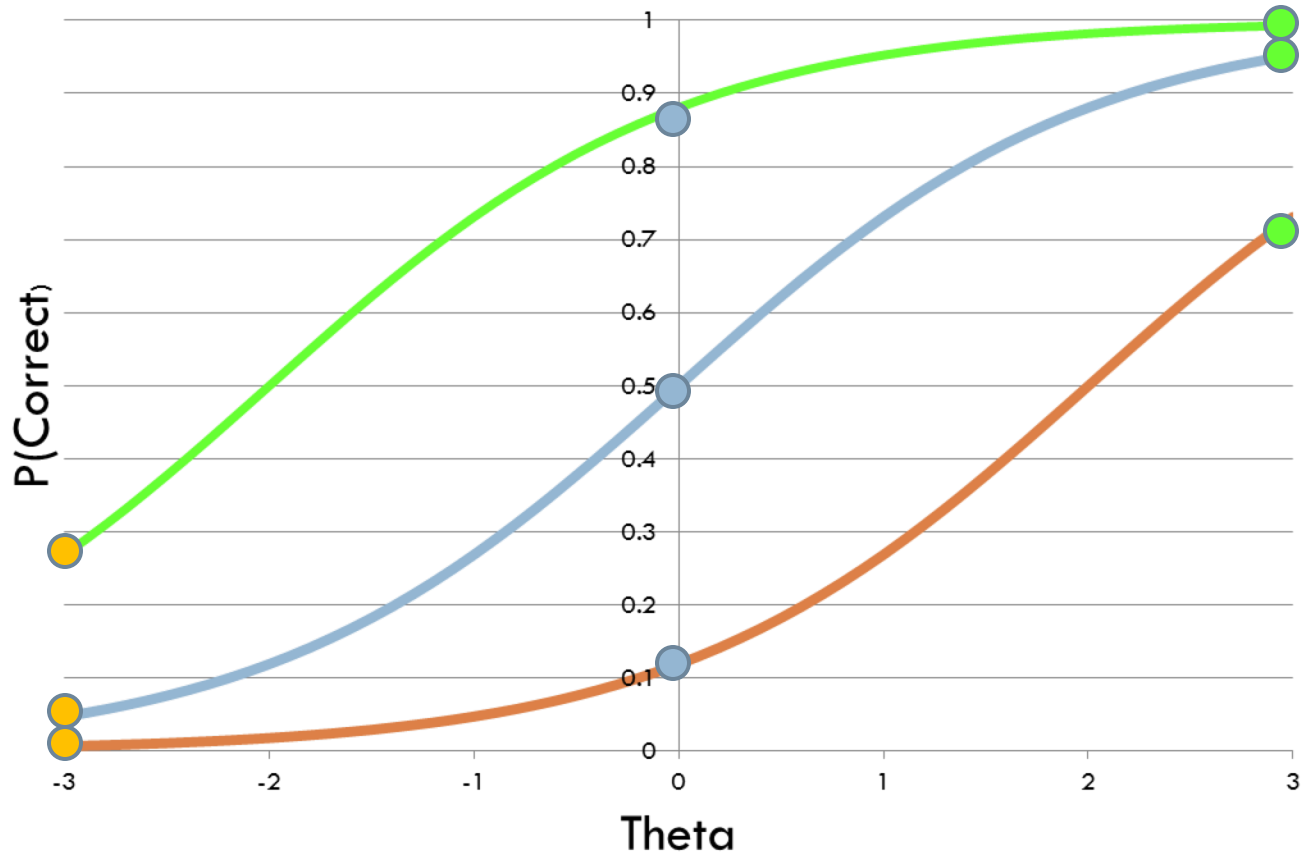
# As student skill goes up, correctness goes up



# Changing difficulty parameter

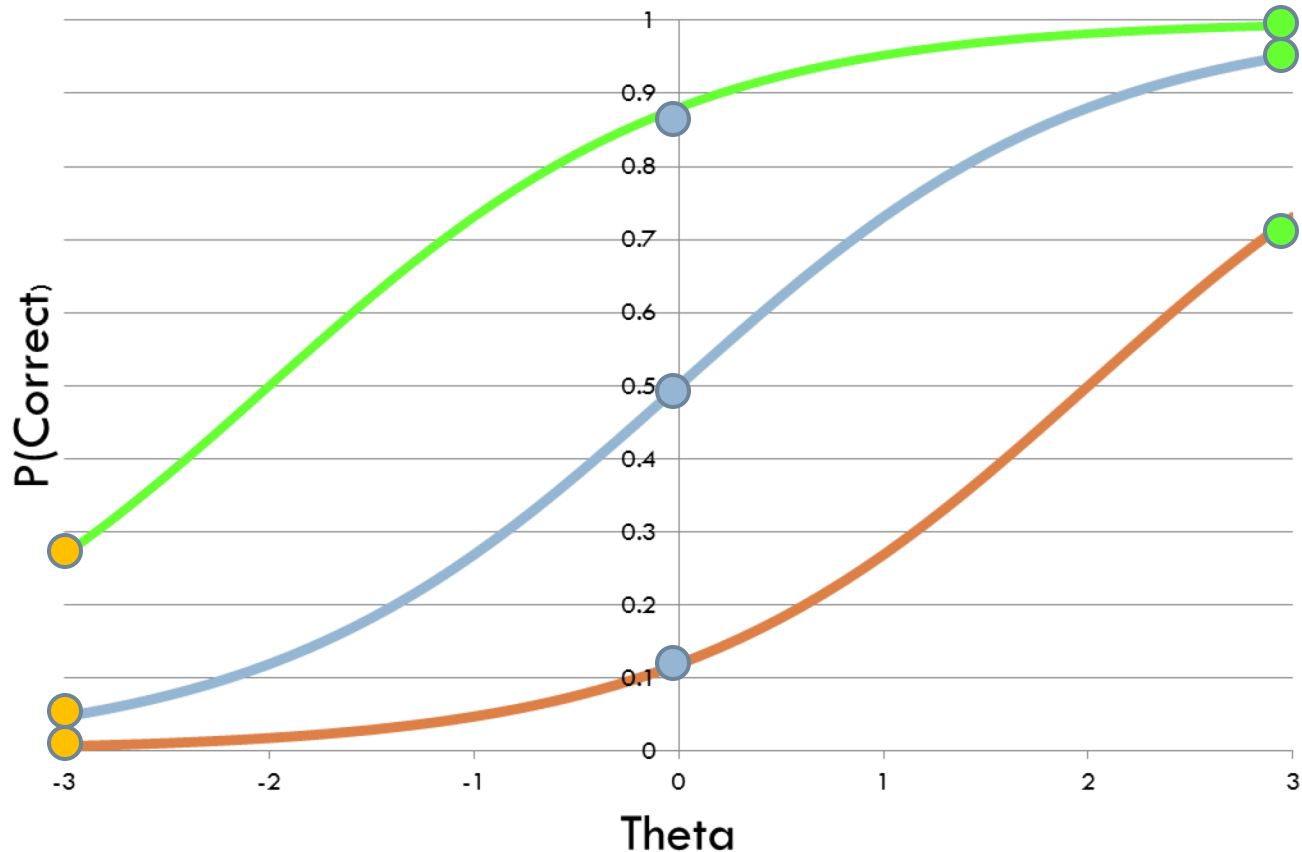
Green line:  $b=-2$  (easy item)

Orange line:  $b=2$  (hard item)



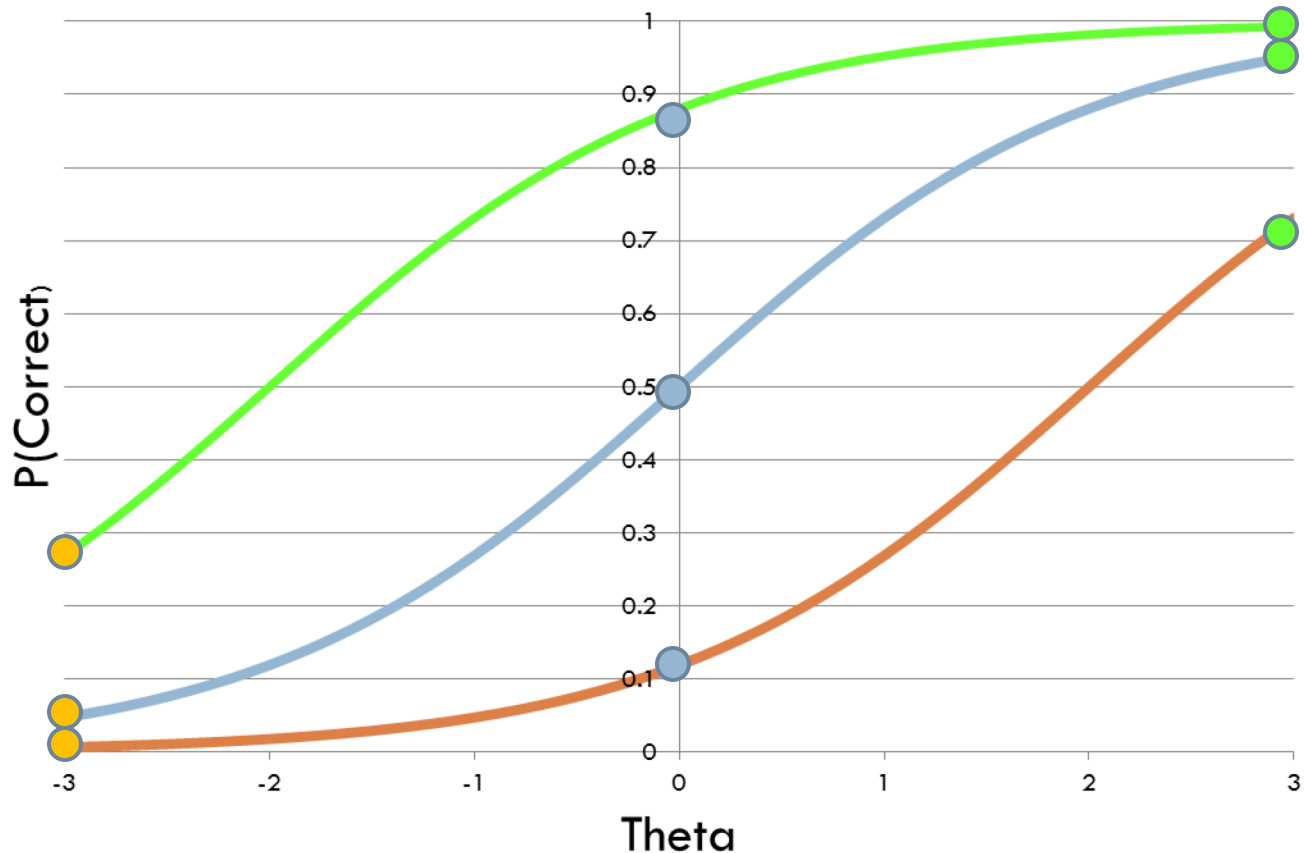
# Note

The good student finds the easy and medium items almost equally difficult



# Note

The weak student finds the medium and hard items almost equally hard

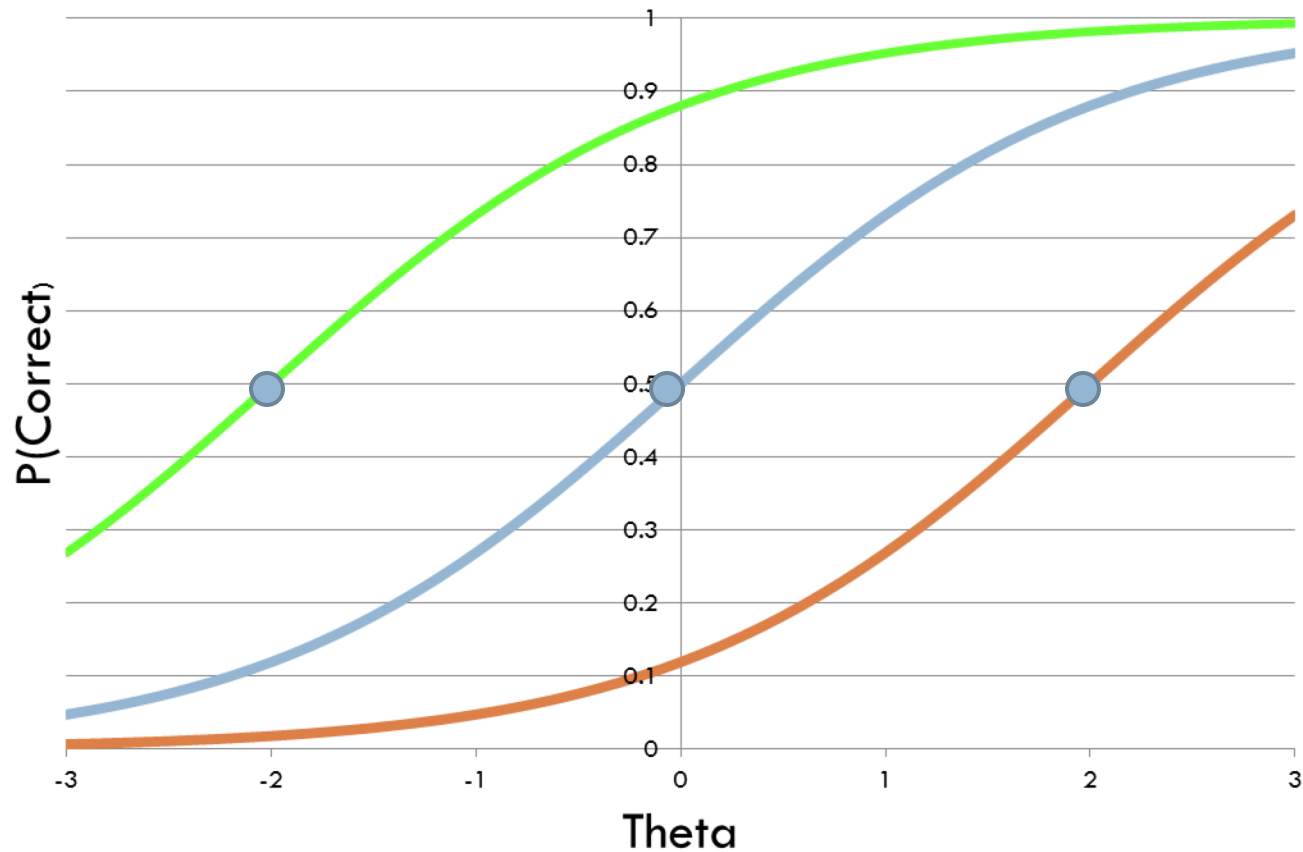




# Note

When  $b=\theta$

Performance is 50%



# The 2PL model

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Another simple IRT model, very popular

Discriminability parameter  $a$  added

$$P(\theta) = \frac{1}{1 + e^{-1(\theta-b)}}$$

Rasch

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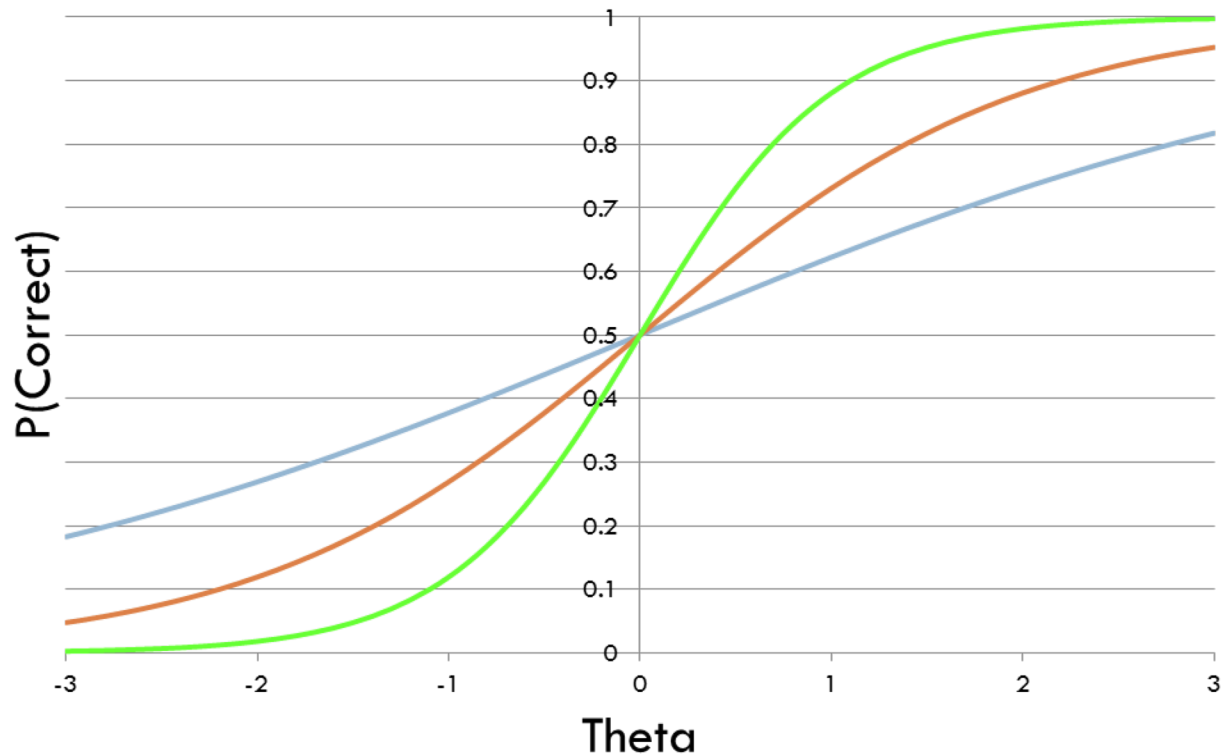
$$P(\theta) = \frac{1}{1 + e^{-a(\theta-b)}}$$

2PL

# Different values of $a$

Green line:  $a = 2$  (higher discriminability)

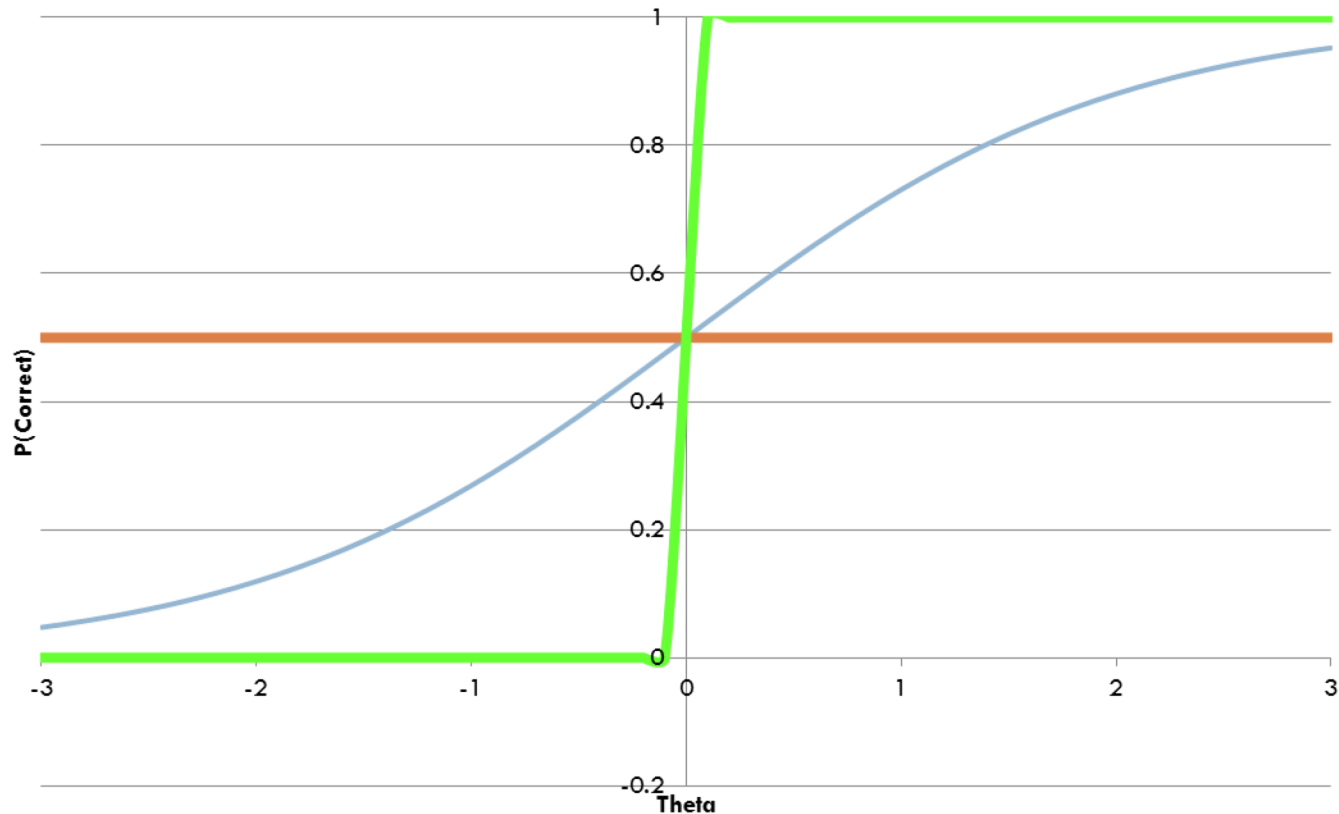
Blue line:  $a = 0.5$  (lower discriminability)



# Extremely high and low discriminability

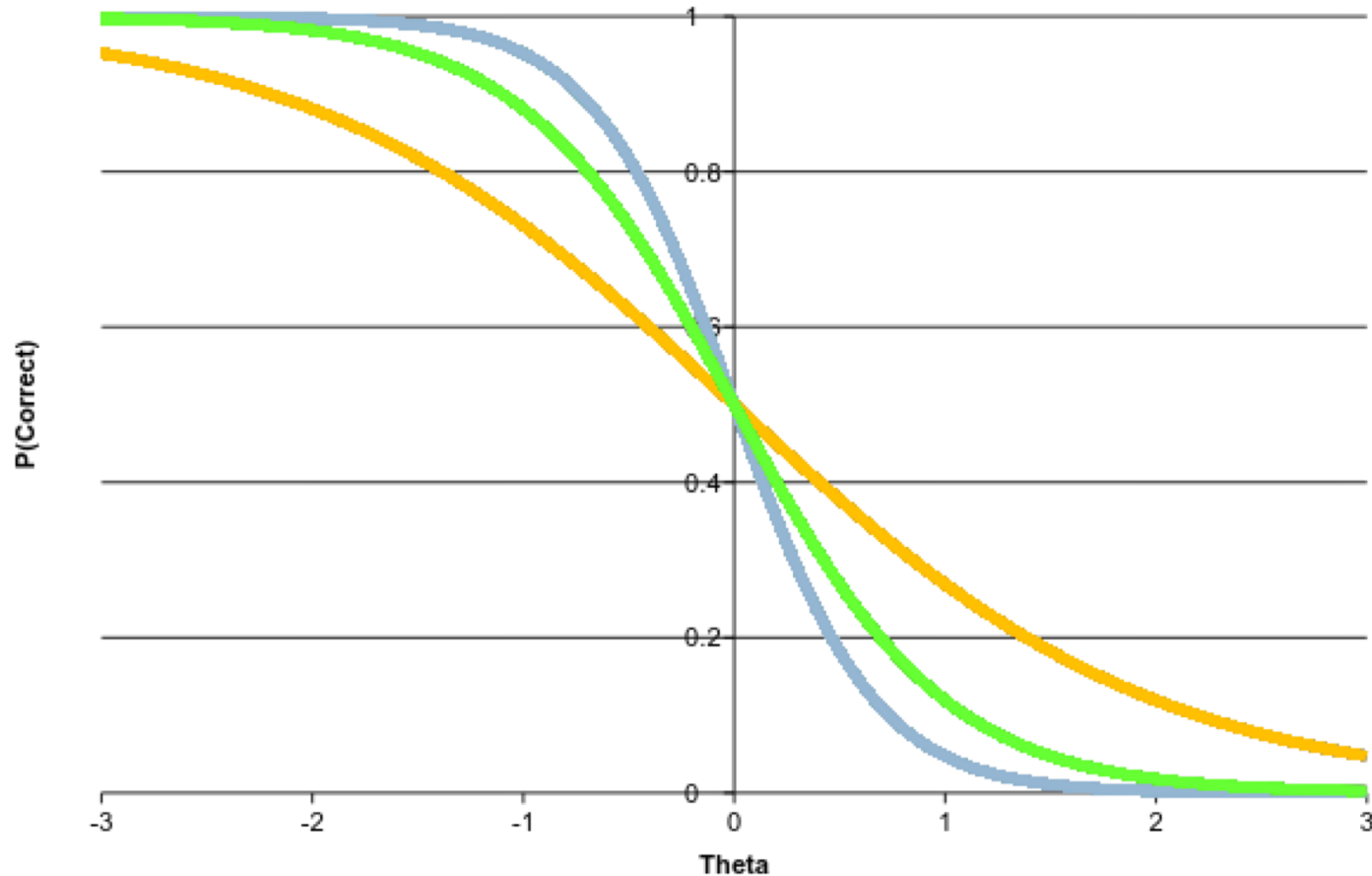
$a=0$

$a$  approaches infinity



# Model degeneracy

a below 0...



# The 3PL model

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A more complex model

Adds a guessing parameter  $c$

# The 3PL model

$$P(\theta) = c + (1 - c) \frac{1}{1 + e^{-a(\theta - b)}}$$

Either you guess (and get it right)

Or you don't guess (and get it right based on knowledge)



# Fitting an IRT model

Can be done with Expectation Maximization  
As discussed in previous lectures

Estimate knowledge and difficulty together  
Then, given item difficulty estimates, you can  
assess a student's knowledge in real time

# Uses...

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IRT is used quite a bit in computer-adaptive testing

Not used quite so often in online learning, where student knowledge is changing as we assess it

For those situations, BKT and PFA are more popular

# ELO (Elo, 1978; Pelanek, 2016)

A variant of the Rasch model which can be used in a running system

Continually estimates item difficulty and student ability, updating both every time a student encounters an item

# ELO (Elo, 1978; Pelanek, 2016)

$$\theta_{i+1} = \theta_i + K (c - P(c))$$

$$b_{i+1} = b_i + K (c - P(c))$$

Where  $K$  is a parameter for how strongly the model should consider new information

# Next Up



Advanced BKT