

Knowledge Inference: Item Response Theory

Item Response Theory

- A classic approach for assessment, used for decades in tests and some online learning environments
- In its classical form, has some key limitations that make it less useful for assessment in online learning
 - But variants such as ELO and CDM address some of those limitations

Key goal of IRT

Measuring how much of some latent trait a person has

- How intelligent is Bob?
- How much does Bob know about snorkeling?
 - SnorkelTutor

Typical use of IRT

- Assess a student's current knowledge of topic
 X
- Based on a sequence of items that are dichotomously scored
 - E.g. the student can get a score of 0 or 1 on each item

Key assumptions

 There is only one latent trait or skill being measured per set of items
 This assumption is relaxed in the extension Cognitive Diagnosis Models (CDM) (Henson, Templin, & Willse, 2009)

No learning is occurring in between items

• E.g. a testing situation with no help or feedback

Key assumptions

- Each learner has ability θ
- Each item has difficulty b and discriminability a
- From these parameters, we can compute the probability P(θ) that the learner will get the item correct



The assumption that all items tap the same latent construct, but have different difficulties

 Is a very different assumption than is seen in PFA or BKT

The Rasch (1PL) model

Simplest IRT model, very popular

- Mathematically the same model (with a different coefficient), but some different practices surrounding the math (that are out of scope for this course)
- There is an entire special interest group of AERA devoted solely to the Rasch model (RaschSIG) and modeling related to Rasch

The Rasch (1PL) model

No discriminability parameter

 Parameters for student ability and item difficulty

The Rasch (1PL) model

Each learner has ability θ

Each item has difficulty b



Item Characteristic Curve

A visualization that shows the relationship between student skill and performance



As student skill goes up, correctness goes up

- This graph represents b=0
- When θ=b (knowledge=difficulty), performance = 50%



As student skill goes up, correctness goes up



Changing difficulty parameter

- Green line: b=-2 (easy item)
- Orange line: b=2 (hard item)





The good student finds the easy and medium items almost equally difficult





The weak student finds the medium and hard items almost equally hard



Note

- When b=θ
- Performance is 50%



The 2PL model

Another simple IRT model, very popular

Discriminability parameter a added

$$P(\theta) = \frac{1}{1 + e^{-1(\theta - b)}}$$
Rasch
$$P(\theta) = \frac{1}{1 + e^{-a(\theta - b)}}$$
2PL

Different values of a

Green line: a = 2 (higher discriminability)
 Blue line: a = 0.5 (lower discriminability)



Extremely high and low discriminability

□ a=0

a approaches infinity



Model degeneracy

□ a below 0...



The 3PL model

A more complex model

Adds a guessing parameter c

The 3PL model

$P(\theta) = c + (1 - c) \frac{1}{1 + e^{-a(\theta - b)}}$

- Either you guess (and get it right)
- Or you don't guess (and get it right based on knowledge)

Fitting an IRT model

Can be done with Expectation Maximization

- As discussed in previous lectures
- Estimate knowledge and difficulty together
 - Then, given item difficulty estimates, you can assess a student's knowledge in real time



IRT is used quite a bit in computer-adaptive testing

- Not used quite so often in online learning, where student knowledge is changing as we assess it
 - For those situations, BKT and PFA are more popular

ELO (Elo, 1978; Pelanek, 2016)

A variant of the Rasch model which can be used in a running system

 Continually estimates item difficulty and student ability, updating both every time a student encounters an item

ELO (Elo, 1978; Pelanek, 2016)

$$\theta_{i+1} = \theta_i + K \left(c - P(c) \right)$$

$$b_{i+1} = b_i + K (c - P(c))$$

Where K is a parameter for how strongly the model should consider new information



Advanced BKT