Week 5 Video 3

Relationship Mining

Association Rule Mining

Association Rule Mining

- Try to automatically find simple if-then rules within the data set

Example

- Famous (and fake) example:
  - People who buy more diapers buy more beer

- If person X buys diapers,
- Person X buys beer

- Conclusion: put expensive beer next to the diapers

Interpretation #1

- Guys are sent to the grocery store to buy diapers, they want to have a drink down at the pub, but they buy beer to get drunk at home instead

Interpretation #2

- There's just no ***time*** to go to the bathroom during a major drinking bout

Serious Issue

- Association rules imply causality by their if-then nature
- But causality can go either direction

If-conditions can be more complex

- If person X buys diapers, and person X is male, and it is after 7pm, then person Y buys beer

Then-conditions can also be more complex

- If person X buys diapers, and person X is male, and it is after 7pm, then person Y buys beer and tortilla chips and salsa
- Can be harder to use, sometimes eliminated from consideration

Useful for…

- Generating hypotheses to study further
- Finding unexpected connections
  - Is there a surprisingly ineffective instructor or math problem?
  - Are there e-learning resources that tend to be selected together?

Association Rule Mining

- Find rules
- Evaluate rules

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Rule Evaluation

- What would make a rule “good”?

Rule Evaluation

- Support/Coverage
- Confidence
- “Interestingness”

Support/Coverage

- Number of data points that fit the rule, divided by the total number of data points
- (Variant: just the number of data points that fit the rule)

Example

- Rule:
  If a student took Advanced Data Mining, the student took Intro Statistics

- Support/coverage?

Took Adv. DM	Took Intro Stat.
1	1
0	1
0	1
0	1
0	1
0	1
1	0
1	0
1	0
1	0
1	1

Example

- Rule:
  If a student took Advanced Data Mining, the student took Intro Statistics

- Support/coverage?
- 2/11= 0.1818

Took Adv. DM	Took Intro Stat.
1	1
0	1
0	1
0	1
0	1
0	1
1	0
1	0
1	0
1	0
1	1

Confidence

- Number of data points that fit the rule, divided by the number of data points that fit the rule’s IF condition

- Equivalent to precision in classification

- Also referred to as accuracy, just to make things confusing
- **NOT** equivalent to accuracy in classification

Example

- Rule:
  If a student took Advanced Data Mining, the student took Intro Statistics

- Confidence?

Took Adv. DM	Took Intro Stat.
1	1
0	1
0	1
0	1
0	1
0	1
1	0
1	0
1	0
1	0
1	1

Example

- Rule:
  If a student took Advanced Data Mining, the student took Intro Statistics
- Confidence?
- 2/6 = 0.33

Took Adv. DM	Took Intro Stat.
1	1
0	1
0	1
0	1
0	1
0	1
1	0
1	0
1	0
1	0
1	1

Important Note

- Implementations of Association Rule Mining sometimes differ based on whether the values for support and confidence (and other metrics)

- Are calculated based on exact cases

- Or some other grouping variable (sometimes called “customer” in specific packages)

For example

- Let's say you are looking at whether boredom follows frustration

- If Frustrated at time N,
  Then Bored at time N+1

Frustrated Time N	Bored Time N+1
0	0
0	0
0	0
0	0
0	0
0	1
1	1
1	1
1	1
1	0
1	1

For example

- If you just calculate it this way,
- Confidence = 4/5

Frustrated Time N	Bored Time N+1
0	0
0	0
0	0
0	0
0	0
0	1
1	1
1	1
1	1
1	0
1	1

For example

- But if you treat student as your "customer" grouping variable

- Then whole rule applies for A, C
- And IF applies for A, C

- So confidence = 1

Student	Frustrated Time N	Bored Time N+1
A	0	0
B	0	0
C	0	0
A	0	0
B	0	0
C	0	1
A	1	1
C	1	1
C	1	1
A	1	0
C	1	1

Arbitrary Cut-offs

- The association rule mining community differs from most other methodological communities by acknowledging that cut-offs for support and confidence are arbitrary
- Researchers typically adjust them to find a desirable number of rules to investigate, ordering from best-to-worst…
- Rather than arbitrarily saying that all rules over a certain cut-off are “good”

Other Metrics

- Support and confidence aren’t enough

- Why not?

Why not?

- Possible to generate large numbers of trivial associations
  - Students who took a course took its prerequisites (AUTHORS REDACTED, 2009)
  - Students who do poorly on the exams fail the course (AUTHOR REDACTED, 2009)

Interestingness

Interestingness

- Not quite what it sounds like
- Typically defined as measures other than support and confidence

- Rather than an actual measure of the novelty or usefulness of the discovery

Potential Interestingness Measures

- Cosine

$$\frac{P(A\text{^}B)}{\mathrm{sqrt}(P(A)*P(B))}$$

- Measures co-occurrence
- Merceron & Yacef (2008) note that it is easy to interpret (numbers closer to 1 than 0 are better; over 0.65 is desirable)


Potential Interestingness Measures

- Lift

$$\frac{\text{Confidence(A->B)}}{\text{P(B)}} = \frac{\text{P(A^B)}}{\text{P(A)*P(B)}}$$

- Measures whether data points that have both A and B are more common than would be expected from the base rate of each
- Merceron & Yacef (2008) note that it is easy to interpret (lift over 1 indicates stronger association)

Merceron & Yacef recommendation

- Rules with high cosine *or* high lift should be considered interesting

Other Interestingness measures

(Tan, Kumar, & Srivastava, 2002)

0.8

1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j) \log \frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))}$
8	J-Measure (J)	$\max\left(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),\right.$ $\left.P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\right)$
9	Gini index (G)	$\max\left(P(A)[P(B\|A)^2 + P(\overline{B}\|A)^2] + P(\overline{A})[P(B\|\overline{A})^2 + P(\overline{B}\|\overline{A})^2] -P(B)^2 - P(\overline{B})^2,\right.$ $\left.P(B)[P(A\|B)^2 + P(\overline{A}\|B)^2] + P(\overline{B})[P(A\|\overline{B})^2 + P(\overline{A}\|\overline{B})^2] -P(A)^2 - P(\overline{A})^2\right)$
12	Laplace (L)	$\max\left(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\right)$
13	Conviction (V)	$\max\left(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B) - P(A)P(B)$
17	Certainty factor (F)	$\max\left(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\right)$
18	Added Value (AV)	$\max(P(B\|A) - P(B), P(A\|B) - P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A) - P(B), P(A\|B) - P(A))$

Worth drawing your attention to

- Jaccard

$$\frac{P(A \wedge B)}{P(A)+P(B)- P(A \wedge B)}$$

- Measures the relative degree to which having A and B together is more likely than having either A or B but not both

Other idea for selection

- Select rules based both on interestingness and based on being different from other rules already selected (e.g., involve different operators)

Alternate approach (Bazaldua et al., 2014)

- Compared "interestingness" measures to human judgments about how interesting the rules were

- They found that Jaccard and Cosine were the best single predictors
- And that Lift had predictive power independent of them

- But they also found that the correlations between [Jaccard and Cosine] and [human ratings of interestingness] were negative
  - For Cosine, opposite of prediction in Merceron & Yacef!

Open debate in the field…

Association Rule Mining

- Find rules
- Evaluate rules

The Apriori algorithm (Agrawal et al., 1996)

1. Generate frequent itemset
2. Generate rules from frequent itemset

Generate Frequent Itemset

- Generate all single items, take those with support over threshold – {i1}
- Generate all pairs of items from items in {i1}, take those with support over threshold – {i2}
- Generate all triplets of items from items in {i2}, take those with support over threshold – {i3}
- And so on…
- Then form joint itemset of all itemsets

Generate Rules From Frequent Itemset

- Given a frequent itemset, take all items with at least two components
- Generate rules from these items
  - E.g. {A,B,C,D} leads to {A,B,C}->D, {A,B,D}->C, {A,B}->{C,D}, etc. etc.
- Eliminate rules with confidence below threshold

Finally

- Rank the resulting rules using your interest measures

Other Algorithms

- Typically differ primarily in terms of style of search for rules

Variant on association rules

- Negative association rules (Brin et al., 1997)
  - What ***doesn't*** go together?
    (especially if probability suggests that two things should go together)
  - People who buy diapers don't buy car wax, even though 30-year old males buy both?
  - People who take advanced data mining don't take hierarchical linear models, even though everyone who takes either has advanced math?
  - Students who game the system don't go off-task?

Next lecture

- Sequential Pattern Mining