

Week 7 Video 6

Knowledge Inference: Q-Matrix

What is the Q-Matrix?

- (Has nothing to do with Keanu Reeves)

What is the Q-Matrix?

- A table
- Where rows are items
- And columns are skills

- (Tatsuoka, 1983; Barnes, 2005)

- Also called a KC [knowledge component] Model
- Or a skill-item mapping

What is the Q-Matrix?

(Tatsuoka, 1983; Barnes, 2005)

	Skill1	Skill2	Skill3	Skill4
Item1	1	0	0	0
Item2	1	1	0	0
Item3	1	0	1	0
Item4	0	0	0	1
Item5	0	0	1	1
Item6	0	1	0	0

Example

	Add	Subtract	Multiply	Divide
$7 + 3 + 2$	1	0	0	0
$7 + 3 - 2$	1	1	0	0
$(7 + 3) * 2$	1	0	1	0
$7 / 3 / 2$	0	0	0	1
$7 * 3 / 2$	0	0	1	1
$7 - 3 - 2$	0	1	0	0

How do we get a skill-item mapping?

- Automatic model discovery
- Hand-development and refinement
- Hybrid approaches

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Automated Model Discovery

- Learn the mapping between items and skills solely from data

One popular algorithm

- Barnes, T., D. Bitzer, & M. Vouk. (2005). Experimental analysis of the q-matrix method in knowledge discovery. *Proceedings of the 15th International Symposium on Methodologies for Intelligent Systems*.



Not the only approach

- Recent interest in non-negative matrix factorization
- Lots of linear algebra
- Desmarais, M.C. (2011) Conditions for effectively deriving a Q-Matrix from data with Non-negative Matrix Factorization. *Proceedings of the 4th International Conference on Educational Data Mining*, 41-50.



First question

- How many skills should we use?

- This is determined empirically
 1. Try 1 skill
 2. Try 1 more skill than previous model (e.g. 2,3,4,5 ...)
 3. Does the new model do better than the previous model?
If so, go to step 2.
If not, quit and use the previous model.

After that: Follow pseudocode

```
Set MinError = LargeNumber;
For Starts= 1 to NumStarts
  Randomly initialize Q[NumCon][NumQues];
  Set Q* = Q; Set CurrError = Error(Q) ;
  For Iter = 1 to NumIter;
    For c= 1 to NumCon
      For q= 1 to NumQues
        Q*[c][q] = Q[c][q] + Delta;
        If (Error(Q*) < CurrError)
          Do
            Set Q=Q*; Set CurrError = Error(Q*);
            Q*[c][q] = Q[c][q] + Delta;
            While (Error(Q*) < CurrError);
          Else
            Q*[c][q] = Q[c][q] - Delta;
            While (Error(Q*) < CurrError)
              Set Q=Q*; Set CurrError = Error(Q*);
              Q*[c][q] = Q[c][q] - Delta;
        If (CurrError < MinError)
          Set BestQ = Q; Set MinError = CurrError;
```

Any questions?



Let's Break That Down

- For each number of skills, the algorithm will be run a certain number of times, with a different (random) initial assignment of items to skills
- This avoids local minima

First Random Version

	Skill 1	Skill 2	Skill 3	Skill 4
$7 + 3 + 2$	0	0	0	0
$7 + 3 - 2$	1	1	0	1
$(7 + 3) * 2$	1	1	0	0
$7 / 3 / 2$	0	1	0	1
$7 * 3 / 2$	1	0	1	1
$7 - 3 - 2$	0	1	0	1

Second Random Version

	Skill 1	Skill 2	Skill 3	Skill 4
$7 + 3 + 2$	1	1	1	0
$7 + 3 - 2$	1	1	1	1
$(7 + 3) * 2$	0	0	0	1
$7 / 3 / 2$	0	1	0	1
$7 * 3 / 2$	1	1	1	1
$7 - 3 - 2$	1	0	1	0

Next...

- Take a set of passes through the table
- Systematically look at whether flipping each 1 to 0 (and each 0 to 1)
- Produces a better model

- Continue this process a predetermined number of times, or until a pass results in no changes

How do we know if it's a better model?

- Several definitions

Barnes et al.'s definition

- Better models have the property that if a student knows skill X
- And item 1 and item 2 both have skill X
- Then a student who gets item 1 right will be more likely to get item 2 right
 - And item 1 wrong \rightarrow item 2 wrong
 - And item 2 right \rightarrow item 1 right
 - And item 2 wrong \rightarrow item 1 wrong

Barnes et al.'s definition

- Given a skill-item mapping, you can predict, for each combination of skills whether a student should get each item correct or not
- A model's degree of error is based on how many item-student pairs the prediction gets wrong

Subtlety

- Is skill conjunctive? (as in Barnes)
 - You need all relevant skills to get an item right
- Or is it compensatory? (Pardos et al., 2008)
 - Any relevant skill leads to getting an item right

Assumption

- The exact approach in Barnes et al. assumes no learning

Alternate Test of Model Goodness

- Look at student improvement over time
- Fit a model like PFA or BKT from Week 4, and see how well it fits data, given the skill-item mapping
- More on this in a sec

How do we get a Q-Matrix?

- Automatic model discovery
- Hand-development and refinement
- Hybrid approaches

Hand Development and Refinement

- The original way that Q-Matrices were created
- A domain expert creates the Q-Matrix using knowledge engineering

Strategies for Q-Matrix Refinement

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- Try to smooth learning curves
- Look for skills with no apparent learning
- Look for problems with unexpected error rates

Tool for doing this

- Pittsburgh Science of Learning Center DataShop
- <https://pslcdatashop.web.cmu.edu/>

PSLC DATA SHOP
a data analysis service for the learning science community

Geometry Area (1996-97) logged in as kk1u@ANDREW.CMU.EDU

Dataset Info Learning Curve Error Report Performance Profiler Export

Line Graph Step Rollup Table LFA Values

Dataset: Geometry Area (1996-97)
Sample(s): All Data

All Selected Knowledge Components

100
90
80
70
60
50
40
30
20
10
0

Error Rate (%)

opportunity

All Data

Opportunity Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of Observations	59	59	59	58	56	55	48	48	47	45	42	42	41	41	41	40	39	39		
Opportunity Number	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

1/1 selected.

Knowledge Components

deselect all | select all

Geometry

1/1 selected.

Knowledge Components

deselect all | select all

- circle-area
- circle-circumfe...
- circle-diameter
- compose-by-addi...
- compose-by-mult...
- equi-tri-height?
- parallelogram-area
- pentagon-area
- rectangle-area
- square-area

12/12 selected.

My Samples

Shared Samples

All Data

Learning Curve

View By

Knowledge Component

Student

Type

Assistance Score

Error Rate

View Predicted

Opportunity Cutoff

Min Clear

Max Clear

Refresh Graph

Knowledge Component Models

Primary Textbook

All Selected K.C.s

circle-area

rectangle-area

trapezoid-area

Possible to look at learning curves for different skill models

If you treat Geometry Area as a single skill,

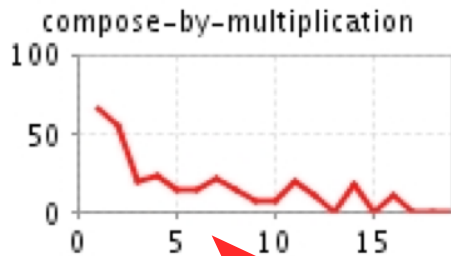
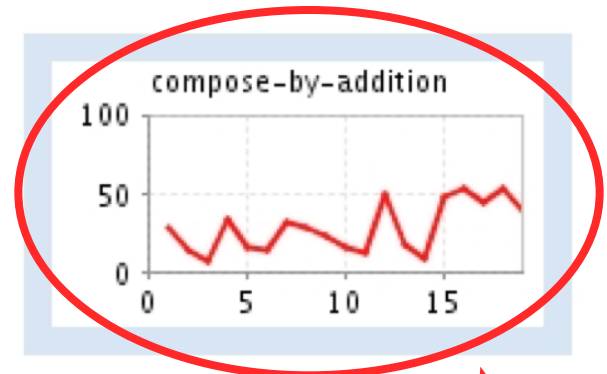
Not a smooth learning curve.

But if you split it into 12 skills

You get a smooth learning curve.

(Rise in error rate because weaker students get assigned more problems)

You can inspect curves for individual skills



Many curves show a reasonable decline (e.g. less errors over time)

Some do not => Opportunity to improve model!

Also look for problems with unexpected error rates

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Shared Samples

- All Data
- PSLC_testdata01
- Stu3110
- Stu3110_a

Performance Profiler

KC Models

Primary: Textbook New

Secondary: DecomposeArith

Knowledge Components

select all | deselect all

- circle-area
- circle-circumfe...
- circle-diameter
- compose-by-addi...
- compose-by-mult...
- equi-tri-height?
- parallelogram-area
- pentagon-area
- trapezoid-area
- triangle-area

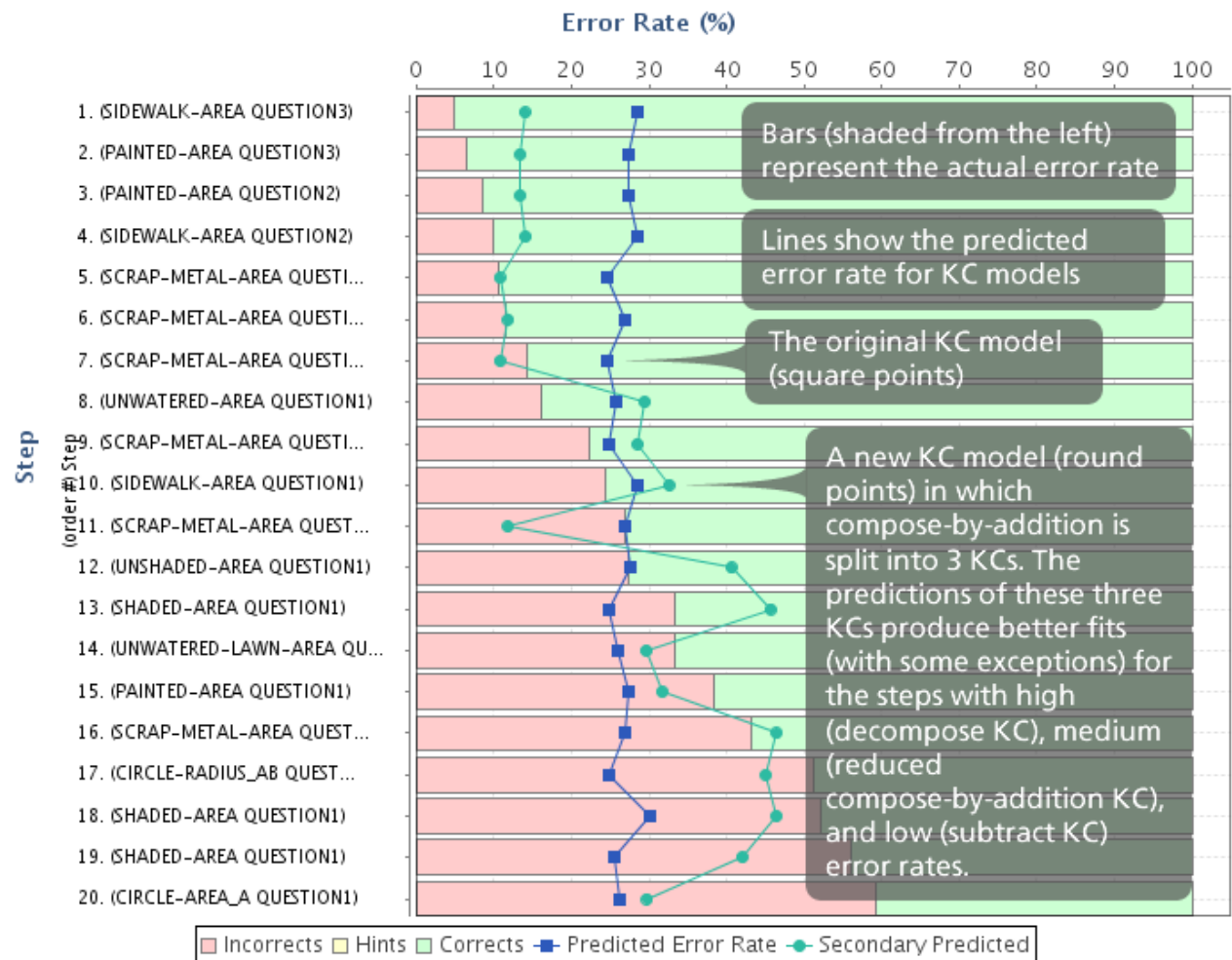
1/10 selected.

Students

59/59 selected.

Problems

40/40 selected.



DataShop can apply model for you!

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- Applies a mathematical model called LFA (similar to PFA) to data
- Can give AIC and BIC goodness measures for different skill-item mappings

Decompose

created by system on 2008-11-21 12:48:05.0

mapping type: correct-transaction-to-kc

LFA values — AIC: 14697.59 BIC: 15237.72

14875 observations labeled with KCs

[show model details](#)

15 KCs

[export](#)

status: ready to use

Textbook

created by system on 2008-11-21 12:48:05.0

mapping type: correct-transaction-to-kc

LFA values — AIC: 14865.38 BIC: 15375.07

14875 observations labeled with KCs

[show model details](#)

13 KCs

[export](#)

status: ready to use

Next Up

- Knowledge Structure Inference: Hybrid Approaches and Models with Prerequisites and Hierarchy