Student Learning Strategies and Behaviors to Predict Success in an Online Adaptive Mathematics Tutoring System

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ABSTRACT
Student learning strategies play a critical role in their overall success. The central goal of this study is to investigate how learning strategies are related to student success in an online adaptive mathematics tutoring system. To accomplish this goal, we developed a model to predict student performance based on their strategies in ALEKS, an online learning environment. We have identified student learning strategies and behaviors in seven main categories: help-seeking, multiple consecutive errors, learning from errors, switching to a new topic, topic mastery, reviewing previous mastered topics, and changes in behavior over time. The model, developed by using stepwise logistic regression, indicated that requesting two consecutive explanations, making an error again after an error and requesting an explanation, and changes in learning behaviors over time, were associated with poorer success in the semester-end assessment. By contrast, the reviewing previous mastered topics strategy was a positive predictor of success in the last assessment. The results showed that the predictive model was able to predict students' success with reasonably high accuracy.

Keywords
Help-seeking, errors, learning strategy, math, student success, adaptive tutoring system

1. INTRODUCTION
Computer-based learning environments, particularly intelligent tutoring systems (ITS), are becoming more commonly used to assist students in their acquisition of knowledge. Computer-based tutors provide tailored instruction and one-to-one tutoring, which can improve students' learning experiences and their motivation. These learning systems also provide unique and critical insight to learning science researchers by creating exhaustive archives of student learning behaviors. A central goal of investigating student learning processes is to unveil the associations between learning behaviors and performance, ultimately allowing learning system developers and researchers to predict and understand student performance. This knowledge allows for evidence-based and individually tailored feedback to be provided to students who are struggling to learn.

2. RELATED WORK
Many studies have investigated the relationships between learning behaviors and success in learning [1, 2, 3]. The most frequent learning behaviors used in the current literature involve help-seeking, making errors, persistence, and changes in learning behaviors over time [4, 5, 6]. For example, worked examples, an effective and commonly used type of help, can be overused by students, negatively affecting learning [7]. However, asking for help after making an error has been found to be an effective help-seeking strategy, particularly for high prior knowledge students [8]. Additionally, reading a worked example after solving a problem can foster better learning than practice alone and reading a worked example before solving a problem can improve learning when compared to reading a worked example after solving a problem [9, 10].

Clearly, there is a delicate interplay between help-seeking strategies students use, their prior knowledge, and learning success. Whether students benefit from making errors often depends on how errors are approached pedagogically. Errors, when treated as stemming from student inadequacies, can trigger math anxiety, which negatively affects students' learning [11, 12, 13]. An extreme example of making errors during learning is seen in wheel-spinning behaviors, in which students attempt ten problems or more without mastering the topic. While too many consecutive errors (i.e. wheel-spinning) undermine learning performance [14], repeated failure in the low-skill phase has been found to improve the likelihood of success in the next step [6] and to lead to more robust learning [15]. Furthermore, the errors that naturally occur from desirable difficulty are considered to be an essential element in learning [16] and facilitate long-term knowledge retention and transfer [17, 18, 19].

Many of the current computer-based tutoring systems are designed to provide students more autonomy, by allowing them to learn at their own pace. In self-paced or self-regulated tutoring systems, students' learning behaviors tend to change over time during learning. These changes in learning behaviors over time represent an important aspect of learning for researchers to understand. Relatively more well-structured behavior over time is positively related to reading performance, whereas more chaotic, less-structured learning behaviors are related to poor reading performance [5].

Persistence is another increasingly studied behavior in learning research. For example, persistence is measured as time spent on unsolved problems during solving anagrams and riddles [20]. Persistence on challenging tasks is associated with mastery goals, which benefit learning [21]. Given these definitions of persistence, a contrasting learning behavior could be considered frequently switching topics within a learning system to find easier topics, an example of gaming the system [22]. Based on students' self-reports, persistence was also found to positively relate to student satisfaction with the computer-based tutoring system [23]. However, unproductive persistence (i.e. wheel-spinning) impedes learning, but various formats of problems and spaced practice can reduce unproductive persistence and improve learning [24].

Reviewing previous learned materials is an efficient way to improve learning. As according to Ebbinghuas' forgetting curve [25], memory retention declines over time. Repeated exposure to previously learned materials can enhance memory retention and improve learning [26]. An example of reviewing previously learned materials is seen in the retrieval practice, which was found to improve students' memory retention of reading materials [27] as well accuracy in solving "student-and-professor" algebra word problems [28].

This study aims to investigate which learning behaviors predict student success in ALEKS (Assessment and Learning Knowledge Spaces), a math tutoring system that adapts to students' knowledge [29]. Given the literature described above, help-seeking behaviors, multiple consecutive errors, learning from errors, temporal behavioral changes, persistence (i.e. switch to a new topic without mastering the current topic), and reviewing previous mastered topics were selected as potential predictors of success in ALEKS. In addition, the percentage of topics that have been mastered, an indicator of learning progress, is included in the model to predict success.

3. Description of ALEKS

ALEKS is a web-based artificially intelligent learning and assessment system [29]. Its artificial intelligence is based on a theoretical framework called Knowledge Space Theory (KST) [30]. KST allows domains to be represented as a knowledge map consisting of a large number of knowledge states, representing the prerequisite relationships between those knowledge states (KS). Therefore, KST allows for a precise description of a student's current knowledge state, and what a student is ready to learn next. ALEKS is able to estimate a student's initial KS by conducting a diagnostic assessment (based on a test) when the student first begins to interact with the system. ALEKS conducts assessments during students' progress through the course to update their knowledge states and to decide what the student is ready to learn next.

In ALEKS, for each topic, a problem is randomly generated, with adjustments made to several parameters for each problem type. This results in an enormous set of unique problems. Students are required to provide solutions in the form of free-response answers, rather than by selecting an answer from multiple choices. Explanations in the form of worked examples can be requested by students at any time. When an explanation is requested, a worked example for the current problem is provided and a new problem is provided to the student. The interface of ALEKS is displayed in Figure 1.

ALEKS is self-paced; students can choose topics to learn and can choose when they want to request help. All the topics that the student is most ready to learn (according to the KST model) are displayed in his or her individual's knowledge pie (Figure 2). The knowledge pie presents the student's learning progress in each math subdomain as well.

Research has shown ALEKS produces learning outcomes comparable with other effective tutoring systems for teaching Algebra [31]. Using ALEKS as an after-school program has also been observed to be as effective as interacting with expert teachers [32]. Students need less assistance during learning when using ALEKS than in traditional curricula [33]. Additionally, ALEKS has been found to reduce the math performance discrepancies between ethnicities in an after-school program [34].

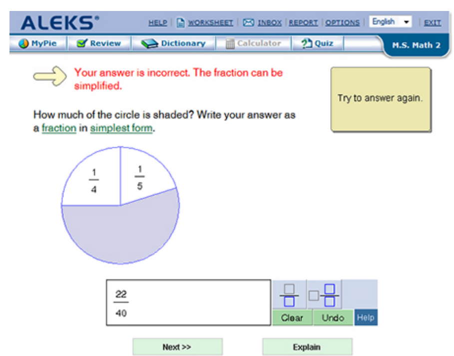

Figure 1. The ALEKS interface

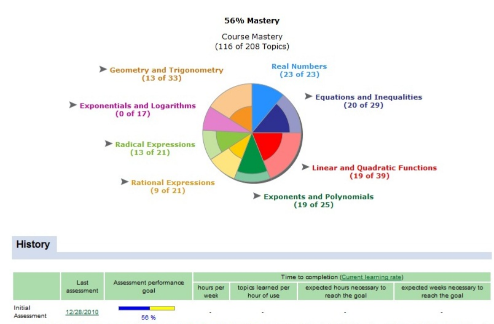

Figure 2. The ALEKS knowledge pie

4. Data

The data used in this study was collected from 179 students within 11 college classes that used ALEKS for developmental mathematics in Fall 2016. The data is comprised of information about students' learning actions and assessment scores. These actions include "correct" (C), "wrong" (W), mastering a topic (S; three C's in a row within a single topic), failing a topic (F; five

W's in a row within a single topic) and explanations (E; requesting an explanation). The data also contains students' last assessment scores in ALEKS which account for students' performance in ALEKS.

5. METHODS

We employed stepwise logistic regression with backward elimination to predict students' success in ALEKS, using a training-test split. More details of this process are described below.

5.1 Student success

Success in ALEKS is defined as students knowing 60% or more of the topics in their last assessment. Therefore, we adopted 60% in the semester-end assessment as a cut-off value for success. Students whose last assessment score was 60% or greater were grouped as "successful students", whereas those with last assessment scores under 60% was grouped as "unsuccessful students". The dataset was randomly split into two parts: 60% of students' data were used to train the model (*N*=107), and 40% were used to test the model's generalizability (*N*= 72). Success was labeled as 1 and failure was labeled as 0 in the prediction model.

5.2 The features to predict success

The following behavior patterns were used to predict student success: (1) help-seeking i.e., requesting an explanation after making an error (WE), and requesting two sequential explanations (EE); (2) multiple consecutive errors i.e., making two sequential errors (WW), making an error again after an error and requesting an explanation (WEW), making an error again after an error and requesting two explanations (WEEW), and the overall percentage of failure labeled by ALEKS (PF); (3) learning from errors i.e., providing a correct answer after making an error (WC), providing a correct answer after making an error and requesting an explanation (WEC), and providing a correct answer after making an error and requesting two explanations (WEEC); (4) switching to a new topic i.e., switching to a new topic after making an error or requesting an explanation (PNew), and switching to a new topic because of failure on a topic (FNew); (5) topic mastery (PS), i.e. providing three correct responses in a row; (6) reviewing previous mastered topics (PReview); and finally, (7) changes in learning behaviors over time (measured using the entropy metric).

The features of the first four aspects mentioned above were generated by using D'Mello's likelihood metric [35] (Equation 1). The likelihood metric is used to compute the transition probability of an event to another event. In the case of multiple events, we calculate a proportion of each sequence out of the number of sequences of that length. For example, the probability of WEEW means the transition probability of WEE to W. In this case, WEE is represented as M_t and W is represented as M_{t+1} in the formula. When the value produced by the likelihood metric is higher than 0, it signifies that M_{t+1} occurs after M_t more frequently than the base rate of M_{t+1}. Otherwise, M_{t+1} occurs after M_t at a rate lower or equal than the base rate of M_{t+1}.

$$L(M_t \rightarrow M_{t+1}) = \frac{\Pr(M_{t+1}|M_t) - \Pr(M_{t+1})}{1 - \Pr(M_{t+1})} \tag{1}$$

Shannon entropy is used to compute the degree of regularity in the changes in students' learning behaviors over time (specifically focusing on the shifts between making an error, give a correct answer, and requesting an explanation) [36] (Equation 2). High entropy values represent disordered leaning behavior patterns. On the contrary, low entropy implies ordered pattern of learning behaviors:

$$H(x) = -\sum_{i=0}^{N} P(x_i)(\log_e P(x_i)) \tag{2}$$

The details on how the features were computed are listed below in table 1.

Table 1. Descriptions of features used to predict success

Features	Description
WE	The transition probability from making an error to requesting an explanation
EE	The transition probability from requesting an explanation to requesting another explanation
WW	The transition probability from making an error to making an error again
WEW	The transition probability from making an error and requesting an explanation to making an error again
WEEW	The transition probability from making an error and requesting two sequential explanations to making an error again
PF	The proportion of times a student made five consecutive errors
WC	The transition probability from making an error to giving a correct answer
WEC	The transition probability from making an error and requesting an explanation to giving a correct answer
WEEC	The transition probability from making an error and requesting two sequential explanations to giving a correct answer
PNew	The probability of starting a new topic after making an error or requesting an explanation on the current topic
FNew	The probability of starting a new topic after failing a topic
PS	The proportion of the mastered topics out of the number of the attempted topics during learning
PReview	The percentage of mastered topics that the student reviews after mastering them
Entropy	The entropy value produced based on students' learning behaviors

6. RESULTS

6.1 Description of features

Before building the prediction model, we calculated basic descriptive statistics. The mean and standard deviations are listed in Table 2.

Table 2. Feature means and standard deviations

Features	M	S.D.
WE	.40	.11
EE	-.07	.07
WW	-.02	.08
WEW	.15	.09
WEEW	.06	.25
PF	.07	.07
WC	-.69	.37
WEC	-.07	.18
WEEC	-.22	.46
PNew	.001	.01
FNew	.76	.33
PS	.87	.10
PReview	.14	.11
Entropy	.51	.11

6.2 Model development

Stepwise logistic regression with backward elimination was used to generate the predictive model of students' success. The final model included requesting an explanation after making an error (WE), requesting two sequential explanations (EE), making an error again after making an error and requesting an explanation (WEW), changes in learning behaviors over time (entropy) and review on the topic (PReview). Each of these metrics were statistically significant predictors of students' success (i.e. the score in the last assessment is greater or less than 60%) in ALEKS. The details on the prediction model are displayed in Table 3.

Table 3. The results of multi-feature logistic regression on students' success

	B	S.E.	Z value	p
Intercept	3.32	1.63	2.04	.04*
WE	4.25	2.31	1.84	.07
EE	-8.31	4.05	-2.06	.04*
WEW	-11.33	3.40	-3.33	.00***
Entropy	-10.34	2.91	-3.55	.00***
PReview	9.44	2.57	3.67	.00***

Note. p<.000 ***, p<.05 *

The results of multicollinearity indicated that there were low correlations between features. The VIF value (i.e. variance inflation factor) for each feature is illustrated in Table 4.

Furthermore, logistic regressions that only include one single feature were conducted to examine suppression effect. The results were listed in Table 5. The results showed that compared to the results of multi-feature logistic regression, the direction of relationship between each feature and success did not change in the single-feature logistic regression. Therefore, the relationship between features and success was not impacted by suppression effect.

Then, based on the results of logistic regressions, students were less likely to be successful in the last assessment if they tend to read two consecutive explanations, or made an error after making an error and requesting an explanation, or demonstrated irregularity in their learning behaviors. By contrast, the more frequently students reviewed topics they have already mastered, the more likely they were to pass the last assessment in ALEKS.

Table 4. Multicollinearity between features in the prediction model

	WE	EE	WEW	entropy	PReview
VIF	1.02	1.32	1.17	1.66	1.50

Table 5. The summary of single-feature logistic regressions on students' success

	B	Z value
WE	4.40	2.32
EE	-0.61	-.23
WEW	-9.12	-3.44
Entropy	-5.01	-2.62
PReview	5.24	2.60

6.3 Model goodness

The fitness index of the prediction model (i.e. AIC) of training data was 115.67. McFadden pseudo r^2 of training data was .30, indicating that this model predicts a substantial amount of the variance in student success.

The model's accuracy of prediction on test data was 0.71. The AUC of test data (area under the ROC curve) was 0.77. The plot of the ROC curve is illustrated in Figure 3.

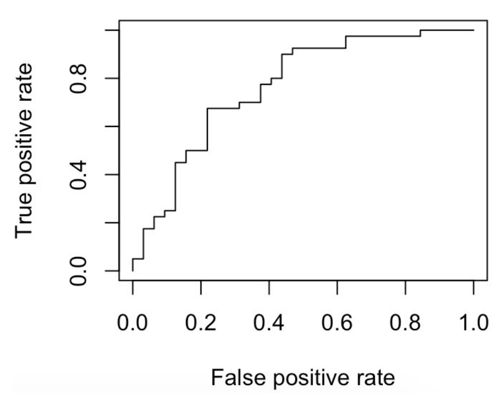

Figure 3. The ROC plot of the prediction model

7. DISCUSSIONS

The current study developed a logistic model to predict student overall success in ALEKS, as well as the relationship between various learning behaviors and success. Our findings contribute to the current understanding of the relationship between student learning behaviors and their delayed performance in adaptive tutoring systems, as well as provide evidence-based suggestions for improving the feedback and interventions in ALEKS.

Requesting two sequential explanations (EE) had a negative relationship with success in the last assessment, a finding in line with previous research on the negative effect of overusing help on learning [9]. However, the EE behaviors may suggest that students did not understand the first explanation rather than indicating that the students were "gaming the system". This can be concluded for the following reason. After requesting a worked-examples explanation, the student typically receives a new problem. Making an error again after making an error and requesting an explanation (WEW) was negatively related to students' success. The relationship between WEW and success suggests that students frequently make multiple consecutive errors, even after receiving the provided worked examples. These students may have trouble understanding the example. Therefore, if students frequently demonstrate those two behaviors on a specific problem, more individually-tailored and deeper-level instructions may be needed to provide the necessary help to overcome the impasse, such as concept-specific conversations with tutor agents that are integrated in ALEKS.

Another finding conforming to the previous research was that regular behaviors during learning is positively related to students' performance [cf. 5]. In this study, the measurement of changes of behaviors over time (via Shannon entropy) is relatively coarse-grained. Moving forward, deeper and finer-grained investigations of changes in behavior over time may shed further light on why regularity is associated with better outcomes.

One other finding worth noting was that the percentage of topics mastered (PS) during learning was not found to be a significant predictor of success on the last assessment. An explanation of this finding may lie in the adaptive design of ALEKS. During learning, ALEKS continually matches students' existing knowledge with topic difficulty and provides the topics that students are most ready to learn, so students focus their time on topics that have an appropriate level of difficulty [24]. Thus, the percentage of topics being mastered may not differ much between students who were successful in the last assessment and those who failed the last assessment. Finally, Reviewing previously mastered topics (PReview) was found to be positively linked to students' success in the last assessment, which confirmed the findings of literature [26].

Our model was able to accurately predict student success. However, some improvements can be made in the future. The current model only includes percentages or probabilities of behaviors without considering the time spent on these behaviors. In the future, adding the time duration of behaviors may increase the prediction accuracy of the model. Additionally, refining the measurements of behaviors may increase the prediction accuracy of the model. For example, changes in learning behaviors over time could be measured during different learning phases or in specific temporal sequences.

By better understanding the factors associated with success in ALEKS, we can design interventions that will improve student success – the ultimate goal of any intelligent tutoring system.

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